MATH 212

Basic Algebra II

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1) Suppose addition and multiplication are defined on $\mathbb{C}^n$, complex n-space, coordinatewise, making $\mathbb{C}^n$ into a ring. Find all ring homomorphisms of $\mathbb{C}^n$ onto $\mathbb{C}$.

2) Show that the additive group of a commutative ring with identity cannot be isomorphic to the additive group $\mathbb{Q}/\mathbb{Z}$.

3) Let R be a principal ideal domain. And let I and J be nonzero ideals in R. Show that $IJ = I \cap J$ if and only if $I + J = R$.

4) Is the ideal of $\mathbb{Z}[X]$ generated by $X^3 + X + 1$ prime?

5) Let I be an ideal and S be a subring of the ring R. Prove that $I \cap S$ is an ideal of S. Give an example to show that every ideal of S need not be of the form $I \cap S$ for some ideal I of R.

6) Let R be a commutative ring and P be a maximal ideal of R. Let $I = P[X]$ be the ideal of the polynomial ring $R[X]$ consisting of the polynomials in $R[X]$ with coefficients in P. Show that I is prime ideal that is not a maximal ideal.

7) Let R be a commutative ring with identity. Let $f(X) = a_0 + a_1X + ... + a_nX^n \in R[X]$. Prove that $f(X)$ is unit if and only if a_0 is a unit in R and a_i is nilpotent for all $i > 0$.

8) Suppose A and B are finitely generated R-modules where R is a principal ideal domain. Show that if $A \oplus A \approx B \oplus B$, then $A \approx B$.

9) Suppose A is a finitely generated R-module where R is a principal ideal domain. If $A \oplus A \approx A$, show that $A \approx 0$.