Lie Algebras HW4

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Throughout L stands for a Lie algebra.

1. Suppose L is nilpotent. Let $0 \neq A \triangleleft L$. Show that $A \cap Z(L) \neq 0$.

2. (Normalizer Condition). Suppose L is nilpotent and $A < L$. Show that $A < N_L(A)$.

3. Show that if $\dim(L) = 2$ then L is solvable of class 2, i.e. $L^{(2)} = [L', L'] = 0$ where $L' = [L, L]$.

4. Show that L' is the smallest ideal I of L such that L/I is abelian.

5. Show that L is nilpotent of class n iff $L^n = 0$ and $L^{n-1} \neq 0$.

6. Show that a solvable Lie algebra has an ideal with trivial multiplication.

7. Let A be such that $L' \leq A \leq L$. Show that $A \triangleleft L$.

8. Show that if L has an ideal of codimension 2 then $L' < L$.

9. Show that $L' < L$ iff L has an ideal of codimension 1.

10. For $X \subseteq L$, set $C_L(X) = \{c \in L : [c, x] = 0 \text{ for all } x \in X\}$. Show that $C_L(X) \leq L$.

11. Show that for "most" of the fields F, $sl_2(F)$ is a simple Lie algebra of dimension 3.