1. Every involution is a product of at most three involutions each of which is a square.

\[(1\ 2) = [(1\ 2)(3\ 4)(5\ 6)(7\ 8)\ ...]\ [(3\ 4)(5\ 6)(7\ 8)(9\ 10)\ ...] = [(1324)(5768)\ ...]^2[(3546)(798\ 10)\ ...]^2\]
\[(12)(34)(56)(78)\ ... = [(1324)(5768)\ ...]^2\]
\[(12)(34)(56) = (1324)^2(56)\]

2. An infinite cycle is a product two involutions each of which is a square.

\[(...75312468...) = [(12)(34)(56)\ ...][[23)(45)(67)\ ...] = [(1324)(5768)\ ...]^2[(2435)(6879)\ ...]^2\]

3. A product of finitely many (finite or infinite) cycles is a product of (finitely many) involutions each of which is a square.

Proof: Enough to do it for finite cycles: \[(123...n) = (12)(23)...(n-1,\ n) = \alpha_1^2\alpha_2^2\ ...\ \alpha_{2n-2}^2\]
where \(\alpha_i\) is an element of order 4 such that \((i, i+1) = \alpha_{2i-1}^2\alpha_{2i}^2\).

4. A product of disjoint
\[(123)(456) = (142536)^2\]

3. Let \(\sigma \in \text{Sym}(N)\). Assume \(\sigma\) is a product of finite disjoint cycles: \(\sigma = \sigma_1\sigma_2\sigma_3\...\)
We may assume that each
\[[(123)(567)(9\ 10\ 11)\ ...][(345)(789)(11\ 12\ 13)\ ...] = (...13\ 9\ 5\ 1\ 2\ 3\ 4\ 6\ 7\ 8\ 10\ 11\ 12\ 14\ ...)\]