1. *Every involution is a product of at most three involutions each of which is a square.*
$(1\ 2) = [(1\ 2)(3\ 4)(5\ 6)(7\ 8)\ ...\ ][(3\ 4)(5\ 6)(7\ 8)(9\ 10)\ ...] = [(1324)(5768)...]^2[(3546)(798\ 10)...]^2$
$(12)(34)(56)(78)... = [(1324)(5768)...]^2$
$(12)(34)(56) = (1324)^2(56)$

2. *An infinite cycle is a product two involutions each of which is a square.*
$(...75312468...) = [(12)(34)(56)\ ...][(23)(45)(67)...] = [(1324)(5768)...]^2[(2435)(6879)...]^2$

3. *A product of finitely many* (*finite or infinite*) *cycles is a product of* (*finitely many*) *involutions each of which is a square.*
Proof: Enough to do it for finite cycles: $(123...n) = (12)(23)...(n-1, n) = \alpha_1^2\alpha_2^2\ ...\ \alpha_{2n-2}^2$
where α_i is an element of order 4 such that $(i, i+1) = \alpha_{2i-1}^2\alpha_{2i}^2$.

4. A product of disjoint
$(123)(456) = (142536)^2$

3. Let $\sigma \in \mathrm{Sym}(\mathbf{N})$. Assume σ is a product of finite disjoint cycles: $\sigma = \sigma_1\sigma_2\sigma_3...$ We may assume that each

$[(123)(567)(9\ 10\ 11)...][(345)(789)(11\ 12\ 13)...] = (...13\ 9\ 5\ 1\ 2\ 3\ 4\ 6\ 7\ 8\ 10\ 11\ 12\ 14\ ...)$