Permutation Groups

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Throughout p denotes a prime natural number.

1. Let A be a normal subgroup of a group G. Any subgroup H of G acts on A by conjugation. What is the kernel of this action?

2. Show that a finite p-group has nontrivial center.

3. Show that $\mathrm{Aut}(\mathbb{Z}/n\mathbb{Z}) \approx (\mathbb{Z}/n\mathbb{Z})^*$.

4. Show that an automorphism of order p of a finite p-group A fixes nontrivial elements of A.

5. Find a group of order p^n for some integer $n > 0$ with an automorphism of order p.

6a. Let G be a group. Define $Z = \{z \in G : zg = gz \text{ for all } g \in G\}$. Show that if G/Z is cyclic then G is abelian.
6b. Show that a group of order p^2 is abelian.
6c. Classify all groups of order p^2. (Hint
6d. Find a nonabelian group of order p^n for $n \geq 3$.
6e. Show that a cyclic group of order p^n ($n \geq 2$) has automorphisms of order p.