

# Group Theory

Math 311

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July 10, 2000

1. Let  $n$  and  $m$  be two integers. Find the isomorphism type of  $\text{End}(\mathbb{Z}/n\mathbb{Z}, \mathbb{Z}/m\mathbb{Z})$ .

2. Let  $G$  be a group generated by  $x_1, \dots, x_n$  and let  $m \in \mathbb{N}$ . Show that the set

$$\left\{ x_{i_1}^{r_1} \dots x_{i_k}^{r_k} : k \in \mathbb{N}, \sum_{j=1}^k r_j \equiv 0 \pmod{m} \right\}$$

is a normal subgroup of index at most  $m$  of  $G$ .

3. Show that  $\text{Sym}(4)$  is solvable but not nilpotent.

4. Classify all abelian groups which satisfy DCC on subgroups.

5. Show that  $\mathbb{Q}^+$  and  $\mathbb{Q}^*$  are not finitely generated.

6. Show that  $\mathbb{Q}^{>0}$  is a free abelian group.

7. Classify all groups of order 6.

8. Show that all groups of order 2000 and 2001 are solvable.