1. Let $H$ and $K$ be two subgroups of $G$. For $x \in G$, the set $HxK$ is called a **double coset** (of $H$ and $K$). Show that the double cosets of $H$ and $K$ partition $G$.

2. For $H \leq G$, let $N_G(H) = \{g \in G : gH = Hg\}$. Show that $H \triangleleft N_G(H) \leq G$ and that $N_G(H)$ is the largest subgroup of $G$ in which $H$ is normal.

3. Let $H$ and $K$ be two subgroups of $G$. Assume $H = \langle X \rangle$ and $Y = \langle Y \rangle$. Show that $\langle H, K \rangle = \langle X, Y \rangle$. **Note:** $\langle X \rangle$ denotes the subgroup generated by $X$ and $\langle X, Y \rangle$ denotes $\langle X \cup Y \rangle$.

4. Let $H$ and $K$ be two subgroups of $G$. Assume $K \triangleleft G$. Show that $\langle H, K \rangle = HK$.

5. Let $\pi$ be a set of primes. A $\pi$-number is an integer whose prime factors are in $\pi$. An element of $G$ whose order is a $\pi$-number is called a $\pi$-element. A group is called a $\pi$-group if all its elements are $\pi$-elements. **5a.** Show that an abelian group generated by $\pi$-elements is a $\pi$-group. **5b.** Show that this is false for nonabelian groups. **5c.** Let $H$ be the subgroup of $G$ generated by all the $\pi$-elements of $G$. Show that for any homomorphism of $\varphi$ of $G$, $\varphi(H) \leq H$. **5d.** Let $H \triangleleft G$ and assume that $H$ and $G/H$ are both $\pi$-groups. Show that $G$ is a $\pi$-group.