Group Theory

Quiz
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G stands for a group throughout.

1. Let H and K be two subgroups of G. For $x \in G$, the set HxK is called a **double coset** (of H and K). Show that the double cosets of H and K partition G.

2. For $H \leq G$, let $N_G(H) = \{g \in G : gH = Hg\}$. Show that $H \lhd N_G(H) \leq G$ and that $N_G(H)$ is the largest subgroup of G in which H is normal.

3. Let H and K be two subgroups of G. Assume $H = \langle X \rangle$ and $Y = \langle Y \rangle$. Show that $\langle H, K \rangle = \langle X, Y \rangle$. **Note:** $\langle X \rangle$ denotes the subgroup generated by X and $\langle X, Y \rangle$ denotes $\langle X \cup Y \rangle$.

4. Let H and K be two subgroups of G. Assume $K \lhd G$. Show that $\langle H, K \rangle = HK$.

5. Let π be a set of primes. A π-number is an integer whose prime factors are in π. An element of G whose order is a π-number is called a π-element. A group is called a π-group if all its elements are π-elements. **5a.** Show that an abelian group generated by π-elements is a π-group. **5b.** Show that this is false for nonabelian groups. **5c.** Let H be the subgroup of G generated by all the π-elements of G. Show that for any homomorphism of φ of G, $\varphi(H) \leq H$. **5d.** Let $H \lhd G$ and assume that H and G/H are both π-groups. Show that G is a π-group.