Math 311

Group Theory
First Midterm
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G always denotes a group.

1. Let H and K be two subgroups of G. Show that for x and y in G, $xH \cap yK$ either is empty or a coset of $H \cap K$.

2. Let H and K be two subgroups of G. An H–K-coset of G is a subset of G of the form HxK for some $x \in G$. Show that the H–K-cosets of G partition G.

3. Show that $C_{\mathrm{Sym}(n)}(1\ 2) \approx \mathbb{Z}/2\mathbb{Z} \times \mathrm{Sym}(n-2)$.

4. Let n be a natural number >1. Prove or disprove for each natural number $n > 1$: A subgroup of index n is normal.

5. Find the isomorphism type of the group (under addition) $\mathrm{End}(\mathbb{Z}/n\mathbb{Z}, \mathbb{Z}/m\mathbb{Z})$.