G always denotes a group.

1. Let $H$ and $K$ be two subgroups of $G$. Show that for $x$ and $y$ in $G$, $xH \cap yK$ either is empty or a coset of $H \cap K$.

2. Let $H$ and $K$ be two subgroups of $G$. An $H$--$K$-coset of $G$ is a subset of $G$ of the form $HxK$ for some $x \in G$. Show that the $H$--$K$-cosets of $G$ partition $G$.

3. Show that $C_{\text{Sym}(n)}(1\ 2) \cong \mathbb{Z}/2\mathbb{Z} \times \text{Sym}(n-2)$.

4. Let $n$ be a natural number >1. Prove or disprove for each natural number $n > 1$: A subgroup of index $n$ is normal.

5. Find the isomorphism type of the group (under addition) $\text{End} (\mathbb{Z}/n\mathbb{Z}, \mathbb{Z}/m\mathbb{Z})$. 