Abstract Mathematics

First Midterm
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1) What is a group?
2) What is an abelian group?
3) Give an example of each of the following:
   a) A finite abelian group,
   b) An infinite abelian group,
   c) A finite nonabelian group,
   d) An infinite nonabelian group.
4) What is a subgroup?
5) What is a normal subgroup?
6) Give examples of nonabelian groups with at least 3 normal subgroups.
7) What is a homomorphism? What is an automorphism? Give examples.
8) Show that if $f\colon G \longrightarrow H$ is a homomorphism, then $f(1) = 1$ and $f(x^{-1}) = f(x)^{-1}$.
9) Recall that if $f\colon G \longrightarrow H$ is a homomorphism, the kernel, Ker(f), is defined as follows:

$$\mathrm{Ker}(f) = \{g \in G : f(g) = 1\}.$$

Show that Ker(f) is a normal subgroup of G.

10) If H is a normal subgroup of G, what can you say about G/H?
11) Find the subgroup of $< \mathbb{Q}, +, 0 >$ generated by 1/2 and 1/5.
12) Find the subgroup of $< \mathbb{Q}^*, \times, 1 >$ generated by all rational numbers > 0 and < 1. (Recall that $\mathbb{Q}^* = \mathbb{Q} \setminus \{0\}$).
13) Find the subgroup of Sym(6) generated by (1,2), (3,4) and (5,6).
14) What is the order of an element in a group?
15) Show that a group whose elements have order 2 is abelian.
16) Let $f\colon G \rightarrow H$ be a homomorphism and $g \in G$. Assume that the order of g and of $f(g)$ are prime to each other. Show that $g = 1$.