Let $G$ be a sharply 2-transitive group acting on a set $X$ of size $n$. Let $x \in X$, $T = G_x$ and $N = (G \setminus \bigcup_{g \in G} T^g) \cup \{1\}$.

1. Show that $|G| = n^2 - n$. (3 pts.)
2. Show that $|T| = n - 1$. (3 pts.)
3. Show that $T^g \cap T = 1$ if $g \not\in T$. (3 pts.)
4. Show that $|N| = n$. (3 pts.)
5. Show that $N \setminus \{1\}$ is the set of elements of $G$ that does not fix a point of $X$. Conclude that $N$ is a normal subset of $G$. (4 pts.)
6. Show that if $n \in N \setminus \{1\}$, then $C_G(n) \subset N$. (4 pts.)
7. Show that $N \setminus \{1\}$ is one conjugacy class. (5 pts.)
8. Find the size of $C_G(n)$ for $n \in N$. (5 pts.)
9. Conclude that $N$ is an abelian group. (5 pts.)
10. Show that $N$ is an elementary abelian group. (7 pts.)
11. Show that $G = N \rtimes T$. (This means that $N$ is a normal subgroup of $G$, $G = NT$ and $N \cap T = \{1\}$). (2 pts.)
12. Show that $G$ has always an involution. (3 pts.)
13. Show that $T$ has an involution iff $n$ is odd. (3 pts.)
14. Show that $N$ has an involution iff $n$ is even. (3 pts.)
15. Show that $T$ has at most one involution, in which case this involution must be central in $T$. (Hint: Assume $T$ has two involutions $i$ and $j$. Let $y \neq x$ and let $g$ carry $(y, iy)$ onto $(y, jy)$. Then $ij^g$ fixes the points $y$ and $iy$ and $g$ fixes $x$ and $y$). (6 pts.)
16. Show that $G$ is a group of exponent some prime.