Math 121
Analysis Retake Corrections

Fall 2012
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1. Let $(a_n)_n$ be a convergent sequence of real numbers. Suppose that $5a_n/2 \in \mathbb{N}$ for all n. What can you say about the sequence $(a_n)_n$ and $\lim_{n\to\infty} a_n$?

2. Decide the convergence of the series

$$\sum_n \frac{1}{\sqrt{|n^2-2|}}, \sum_n \frac{1}{\sqrt{n^2+1}}, \sum_n \frac{1}{\sqrt{|n^4-6|}}.$$

3. Suppose that $(a_n)_n$ is a positive and decreasing sequence and that the series $\sum_n a_n$ is convergent. Show that $\lim_{n\to\infty} na_n = 0$.

4. Find a positive sequence $(a_n)_n$ such that the series $\sum_n a_n$ is convergent but that $\lim_{n\to\infty} na_n \neq 0$.

5. Let $(a_n)_n$ be a sequence. Suppose that $\sum_{n=1}^{\infty} |a_n - a_{n+1}|$ converges. Such a sequence is called of **bounded variation**. Show that a sequence of bounded variation converges.