Algebra I Final

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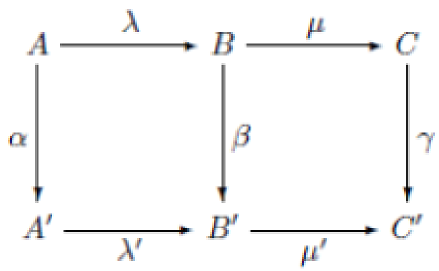
1. Find all groups which have exactly three subgroups.
2. Find all groups which have exactly four subgroups.
3. Find all groups which have exactly five subgroups.
4. Find Aut(D_{2n}). Here D_{2n} is the dihedral group with $2n$ elements.
5. The set of all rotation of $\mathbb{R}$ around O is a group denoted $SO_3(\mathbb{R})$. Show that $SO_3(\mathbb{R})$ is not abelian.
6. Let R be a ring. Let $\wp$ be the set of prime ideals of R. For each subset $X \subseteq R$, let

$$U_X = \{I \in \wp : X \nsubseteq I\}.$$

   Show that $\tau = \{U_X : X \subseteq R\}$ defines a topology on R. Show that this topology is compact.
7. Let R be a ring and



   be a commutative diagram of R-modules and R-module homomorphisms with exact rows (i.e. Im λ = Ker μ and the same for the second row). Show the following:

   a. If α, γ and λ' are monomorphisms then so is β.

   b. If α, γ and μ are epimorphisms then so is β.

   c. (Im $\beta \cap$ Im λ')/Im $(\lambda' \circ \alpha) \approx$ Ker$(\mu' \circ \beta)$/(Ker $\mu \cap$ Ker β).