MATH 113
Self Study Material
Set Theory

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Number Theory

1)(Prime Desert) Show that for all $n \in \mathbb{N}$ there exists A, $B \in \mathbb{N}$ with $A - B \geq n$ such that there is no prime between A and B.

2) There are infinitely many primes of the form $4k + 3$.

3) For a prime p, if $p = 3k + 1$ then $p = 6m + 1$.

4) Define $F_n = 2^{2^n} + 1$. Show that $F_0.F_1...F_{n-1} + 2 = F_n$. Deduce that there are infinitely many primes.

5) Is $2^{13} - 1$ prime ?

Naive Set Theory and Combinatorics

1) Let X be a set. Show that there is no onto function from X to $\mathcal{P}(X)$.

2) Let U be any non-empty set
a) Let $\phi(x)$ and $\psi(x)$ be two properties (of elements of U). Assume that $\phi(x) \Rightarrow \psi(x)$. Look at the sets

$$U_\phi = \{x \in U : \phi(x)\} \quad \textit{and} \quad U_\psi = \{x \in U : \psi(x)\}$$

What is the relationship between U_ϕ and U_ψ ?
b) Take $U = \mathbb{Z}$ and find concrete examples for ϕ, ψ, U_ϕ, and U_ψ.
(Hint: Consider the relation divides in $\mathbb{Z}$).

3) Find a bijection between $\mathbb{N} \times \mathbb{N}$ and $\mathbb{N}$ explicitely.

4) Find an infinite family $(A_i)_{i\in\mathbb{N}}$ of sets so that any finite intersection of these sets is non-empty but $\bigcap\limits_{i\in\mathbb{N}} A_i = \emptyset$. And another infinite family $(B_i)_{i\in\mathbb{N}}$ of sets so that $\bigcap\limits_{i\in\mathbb{N}} B_i$ has only one element.

5) Let $\mathcal{P}$ be the power set of $I = \{1, 2, 3, 4, 5\}$. Let S be a randomly chosen subset of $\mathcal{P}$. What is the probability that S is the power set of some subset of I ?

6) Let $|X| = n$ and $|Y| = m$. Find the number of all bijections from X to X and number of all functions from X to Y and number of all one to one functions from X to Y and number of all onto functions from X to Y.

Axiomatic Set Theory

1) By using $\mathbb{N}$ is well ordered define the succesor function s (Assume you do not know s and define it from the well order of $\mathbb{N}$).

2) Let A and B be two sets. Show that the collection of all functions from A to B is a set.

3) Let X be set and let $\equiv$ be an equivalance relation on X. Define the equivalance classes of X under $\equiv$ as $\overline{x} = \{y \in X : x \equiv y\}$. Show that the collection of all the equivalance classes of X under $\equiv$ is a set.

4) Show that there is no $y \in \mathbb{N}$ such that $x < y < S(x)$.

5) Show that there are no $z \in \mathbb{Z}$ such that $z + z = \overline{(1, 0)}$.

6) Show that for any $a, b \in \mathbb{Q}$ there is an element $c \in \mathbb{Q}$ such that $a < c < b$.

7) Show that there is no $x \in \mathbb{Q}$ such that $x \times x = 2$.