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Homework Problems for Set Theory 2

Sheet 1

1. Prove that any family of disjoint non-empty open inetrvals is finite or countable.

2. (a) Prove that any family of disjoint 8-signs on the plane is at most countable.
   (b) Prove a similar statement for letters `T` and `E`.
   (c) What about `M` and `O`?

3. A point $x \in \mathbb{R}$ is called a maximum point for a function $f : \mathbb{R} \longrightarrow \mathbb{R}$ if there exists some $\varepsilon > 0$ such that $f(x) > f(x+h)$ for all $|h| < \varepsilon$. Prove that the set of all maximum points for any function $f : \mathbb{R} \longrightarrow \mathbb{R}$ is at most countable.

4. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a nondecreasing function. Prove that f is continuous everywhere except for some countable set.

5. Construct a one-to-one correspondence between $[0, 1]$ and $[0, 1)$.

6. Suppose A is infinite and uncountable. If B is finite or countable, show that $A - B$ and A havethe same cardinality.

7. Prove that a set A is infinite iff there exists a one-to-one correspondence between A and a proper subset B of A.

8. Construct a one-to-one correspondence between the set $[0, 1] \cup [2, 3] \cup [4, 5] \cup \cdots$ and $[0, 1]$.

9. Prove that the set of all points in the plane has the same cardinality as the set of all lines.

10. Prove the following formulas:
    (a) $(A \times B) \simeq (B \times A)$, $[(A \times B) \times C] \simeq [A \times (B \times C)]$.
    (b) $A \simeq C, B \simeq D \implies (A \times B) \simeq (C \times D)$.
    (c) $(A \simeq B) \implies (2^A \simeq 2^B)$.
    (d) $A \simeq C, B \simeq D, A \cap B = \emptyset = C \cap D \implies [(A \cup B) \simeq (C \cup D)]$.