# Analysis I (Math 121)
# Resit – 2

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Justify all your answers. A nonjustified answer will not receive any grade whatsoever, even if the answer is correct.

**DO NOT** use symbols such as $\forall$, $\exists$, $\Rightarrow$. Any text that uses such symbols will be ignored and will not be graded.

**Make full sentences** with correct punctuation. You may write in Turkish or in English. Yani Türkçe yazabilirsiniz.

1. Let $(a_n)_n$ be a converging sequence of nonzero real numbers. Do the sequences $\left(\frac{a_n}{a_{n+1}}\right)_n$, $\left(\frac{1-a_n}{1-a_{n+1}}\right)_n$ and $\left(\frac{a_n}{1+a_{n+1}^2}\right)_n$ converge? (6 pts.)

2. Let $(a_n)_n$ be a convergent sequence of natural numbers. Is it true that $\lim_{n\to\infty} a_n \in \mathbb{N}$? (4 pts.)

3. Let $x > 1$. Discuss the convergence of $(x^n/n!)_n$ (4 pts.)

4. Let $x \in \mathbb{R}$. Discuss the convergence of $(x^{n!}/n!)_n$ (8 pts.)

5. Let $(a_n)_n$ be a convergent sequence of real numbers. Suppose that $\lim_{n\to\infty} a_n \in \mathbb{Q}$. Is it true that $a_n \in \mathbb{Q}$ for infinitely many $n$? (3 pts.)

6. Let $(a_n)_n$ be a sequence of real numbers such that the subsequences $(a_{2n})_n$, $(a_{2n+1})_n$ and $(a_{7n+1})_n$ all converge. Does the sequence $(a_n)_n$ converge necessarily? (5 pts.)

7. Let $(a_n)_n$ be a sequence of real numbers such that the sequence $(a_n^2)_n$ converges. Discuss the convergence of the sequence $(a_n)_n$. (5 pts.)

8. Let $(a_n)_n$ be a sequence of real numbers such that $\lim_{n\to\infty} a_n = \infty$. Is it true that $\lim_{n\to\infty} a_{2n} = \infty$? (2 pts.)

9. Let $(a_n)_n$ be a sequence of real numbers such that $\lim_{n\to\infty} a_n = \infty$. Discuss the convergence of $\lim_{n\to\infty} 1/a_{2n} = 0$. (10 pts.)

10. Find the following limits and prove your result using **only** the definition. (18 pts.)

a. $lim_{n\to\infty} \frac{3n^2-5}{5n^2+3n}$.

b. $lim_{n\to\infty} \frac{n^2-5n+3}{-4n+2}$

c. $lim_{n\to\infty} \frac{5n^2-4}{2n^3+2}$

11. Find $lim_{n\to\infty} \left(\frac{n^2-1}{n^3-n-5}\right)^{\frac{n^2-1}{2n-3}}$. (10 pts. Justify your answer).

12. Show that the series $\sum_{n=0}^{\infty}(-1)^n z^{2n+1}/(2n+1)!$ converges for all $z \in \mathbb{R}$. (4 pts.)

13. Show that the series $\sum_{n=1}^{\infty} \left(\frac{n}{n^2+1}\right)^{n/3}$ converges. Find an upper bound for the sum. (10 pts.)

14. Let $(a_n)_n$ be a sequence of real numbers. Assume that there is an $r > 1$ such that $|a_{n+1}| \geq r|a_n|$ for all $n$. What can you say about the convergence or the divergence of $(a_n)_n$? (6 pts.)