Final
Math 120B (Fall 1994)

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1. Find all elements of order 5, 6 and 7 of Sym(5).

2. Let G be a group and H be a subgroup of G. Show that the map

$$gH \longmapsto Hg^{-1}$$

is a bijection between the set of left cosets of H in G and the set of right cosets of H in G.

3. Let G be a group and H a normal subgroup of G.

3a. Show that the map

$$g \longmapsto gH$$

is a group homomorphism from G onto G/H.

3b. What is the kernel of the above homomorphism?

4. Let $\phi : G \longrightarrow H$ be a surjective homomorphism between two groups. Let K be a normal subgroup of G. Show that $\phi(K)$ is a normal subgroup of H.

5. Find all homomorphisms from $\mathbb{Z}$ into itself.

6. Let $G := \{\begin{pmatrix} x & y \\ z & t \end{pmatrix} : x, y, z, t \in \mathbb{Z} \text{ and } xt - yz = 1\}$.

6a. Is G a group? Why? What is the inverse of the element $\begin{pmatrix} x & y \\ z & t \end{pmatrix}$ of G?

6b. For an integer n, define

$$G_n := \{\begin{pmatrix} x & y \\ z & t \end{pmatrix} \in G : x, t \in 1 + n\mathbb{Z} \text{ and } y, z \in n\mathbb{Z}\}.$$

Show that G_n is a normal subgroup of G and that G/G_n is finite.