Topology

Summer 2002

1. Let X be a topological space and $Y \subseteq X$ be a subset. What is the the smallest topology on Y that makes the cannonical embedding $i : Y \to X$ continuous?

2. For each $i \in I$ let X_i be a topological space. What is the the smallest topology on $\prod_{i\in I} X_i$ that makes all the projection maps $\pi_j : \prod_{i\in I} X_i \to X_j$ ($j \in I$) continuous?

3. Take $I = \mathbb{N}$ and $X_i = \mathbb{N}$ with the discrete topology. Is the topology on $\Pi_{i\in\mathbb{N}} \mathbb{N}$ induced from a metric?

Note that $\Pi_{i\in\mathbb{N}} \mathbb{N}$ can be regarded as functions from $\mathbb{N}$ into $\mathbb{N}$.

5. Show that the set of injective maps from $\mathbb{N}$ into $\mathbb{N}$ is a closed set.

6. Show that the set of surjective maps from $\mathbb{N}$ into $\mathbb{N}$ is not a closed set.

5. Show that $\mathrm{Sym}(\mathbb{N})$ is a topological space with respect to the induced metric.