Topology HW15

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1. Let X be a metric space. Show that X has a countable base iff X has a countable dense subset.

2. (Sorgenfrey Line). Let $X = \mathbb{R}$ and consider the topology on $\mathbb{R}$ generated by $\{[a, b) : a, b \in \mathbb{R}\}$.

2a. Is $\mathbb{R}$ (with this topology) connected?

2b. Is $\mathbb{R}$ (with this topology) compact?

2c. Show that $\mathbb{R}$ (with this topology) has a countable base.

2d. Show that $\mathbb{R}$ (with this topology) has no countable dense subset.

2e. Conclude that $\mathbb{R}$ (with this topology) is not metrisable.

3. We say that two topologies τ_1 and τ_2 on the same set X are equivalent if there is a continuous bijection from (X, τ_1) onto (X, τ_2) such that f^{-1} is continuous from (X, τ_2) onto (X, τ_1). How many nonequivalent topologies do you have on a set with three elements?

4. Let X be a topological space. Let $(x_n)_n$ be a sequence in X. Let $x \in X$. We say that x is a limit of $(x_n)_n$ if for every open subset U containing x there is a natural number N such that for all $n > N$, $x_n \in U$.

4a. Find an example of a topological space where a sequence has more than one limit.

4b. Show that if X is a Hausdorff topological space then a sequence has at most one limit.

4c. Is it true that a topological space where every sequence has at most one limit is Hausdorff.