1. Let $X$ be a metric space. Show that $X$ has a countable base iff $X$ has a countable dense subset.

2. (Sorgenfrey Line). Let $X = \mathbb{R}$ and consider the topology on $\mathbb{R}$ generated by $\{(a, b) : a, b \in \mathbb{R}\}$.

   2a. Is $\mathbb{R}$ (with this topology) connected?
   2b. Is $\mathbb{R}$ (with this topology) compact?
   2c. Show that $\mathbb{R}$ (with this topology) has a countable base.
   2d. Show that $\mathbb{R}$ (with this topology) has no countable dense subset.
   2e. Conclude that $\mathbb{R}$ (with this topology) is not metrisable.

3. We say that two topologies $\tau_1$ and $\tau_2$ on the same set $X$ are equivalent if there is a continuous bijection from $(X, \tau_1)$ onto $(X, \tau_2)$ such that $f^{-1}$ is continuous from $(X, \tau_2)$ onto $(X, \tau_1)$. How many nonequivalent topologies do you have on a set with three elements?

4. Let $X$ be a topological space. Let $(x_n)_n$ be a sequence in $X$. Let $x \in X$. We say that $x$ is a limit of $(x_n)_n$ if for every open subset $U$ containing $x$ there is a natural number $N$ such that for all $n > N$, $x_n \in U$.

   4a. Find an example of a topological space where a sequence has more than one limit.
   4b. Show that if $X$ is a Hausdorff topological space then a sequence has at most one limit.
   4c. Is it true that a topological space where every sequence has at most one limit is Hausdorff.