The topological space $X$ is said to be **connected** if $X$ is not the union of two disjoint nonempty open subsets. Note that $X$ is connected if only if $\emptyset$ and $X$ are the only clopen subsets of $X$. A subset $A$ of $X$ is called **connected** if $A$ is a connected space with its induced topology. Clearly, a singleton subset of $X$ is always connected.

1. Is $\mathbb{Q}$ connected?
2. Let $A$ be a connected subset of $X$. Show that if $A \subseteq B \subseteq A$, then $B$ is connected.
3. Show that $\mathbb{R}$ is connected.
4. Let $(A_i)_i$ be a family of connected subsets of $X$. Assume that for $i \neq j$, $A_i \cap A_j \neq \emptyset$. Show that $\bigcup_i A_i$ is connected.
5. Let $(A_i)_i = 0, 1, 2, \ldots$ be a sequence of connected subsets of $X$. Assume that for all $i$, $A_i \cap A_{i+1} \neq \emptyset$. Show that $\bigcup_i A_i$ is connected.
6. Let $A \subseteq X$ be connected. Let $B \subseteq X$. Assume that $A \cap B \neq \emptyset$ and $A \cap B^c \neq \emptyset$. Show that $A \cap \delta B \neq \emptyset$.
7. Deduce from above that if $X$ is connected and $\emptyset \neq A \subset X$, then $\delta A \neq \emptyset$.
8. Let $x \in X$. Show that the union of connected subsets of $X$ that contain $x$ is the largest connected subset of $X$ containing $x$. This subset is called **connected component** of $x$.
9. Show that connected components of points of $x$ partition $X$.
10. Let $X = \mathbb{Q}$. What is connected component of any $q \in \mathbb{Q}$?
11. Find an example where the connected components are not closed.
12. Show that the connected components of a space are closed.
13. Show that if the connected components are open, then they are also closed.
14. Show that if $X$ has finitely many connected components, then each connected component is clopen.
15. Show that the image of a connected set under a continuous map is connected.
16. Show that the product of two connected spaces is connected.