Algebra (Math 211) Final
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1a. How many conjugacy classes are there in Sym(5)?
1b. Find the sizes of the centralizers of the elements of Sym(5).

2a. Find all automorphisms of the additive group of **Z.**
2b. Find all automorphisms of the ring **Q.**

3a. Find a generator of the subgroup of $\mathbf{Q}^*$ generated by 3/5 and 5/7.
3b. Is the subgroup of $\mathbf{Q}^+$ generated by $\{3^n : n \in \mathbf{Z}\}$ cyclic?
3c. Is the subgroup of $\mathbf{Q}^*$ generated by $\{3^n : n \in \mathbf{Z}\}$ cyclic?

4a. Find three elements of order 2 of $\mathbf{R}^*/\langle\sqrt{12}\rangle$.
4b. Is there an element of infinite order in **Q/Z**? Justify your answer.
4c. Is there an element of finite order in **R/Q**? Justify your answer.

5. Show that the set of elements σ of Sym(**Z**) such that $\sigma(0) = 0$ is a subgroup isomorphic to Sym(**Z**).

6. Find a nonabelian group of order 8.

7. Find the invertible elements of **R**[X, Y]/I where I is the principal ideal generated by $X^2 - Y^2$.

8. Let R be a ring (not necessarily commutative, does not necessarily have an identity) where $x^2 = x$ for all $x \in R$. Show that R is commutative and has characteristic 2.

9. Is the ideal generated by $X - Y$ a maximal ideal of **R**[X, Y]? Justify your answer.

10. Find a maximal ideal of **Z**[X, Y]/I where I is the ideal generated by $X^3 - Y^2$.