Topology HW4

Gümüşlük Akademisi
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Özlem Beyarslan

1. Let Y be an open subset in the metric space (X,d) and let x_0 be an element of Y.

1a. Prove that the set $Y \setminus \{x_o\}$ is also open.

1b. Deduce that, if we remove finitely many points from Y, the remaining set is also open. Is the above statement still true if we remove infinitely many points? Prove or give a counterexample.

2. Let X be a topological space. A subset of X is called **closed** if it is the complement of an open set in the topology.

2a. Show that Ø and X are closed subsets.

2b. Show that the intersection of arbitrarily many closed subsets is closed.

2c. Show that if A and B are closed subsets then $A \cup B$ is closed.

2d. Show that in a topology induced by a metric space, a singleton set is closed. Is this true for an arbitrary topology?

2e. Is it true that union of arbitrarily many closed sets is closed? Prove or give counterexample.

3. Give an example of a subset of $\mathbb{R}^2$ which is neighter open nor closed. Prove your assertion.

4. Show that the closed ball with center (0,0) and radius 1 in the usual (Euclidean) topology on $\mathbb{R}^2$ is closed.

5. Assume $U = \{\{x\} : x \in X\}$. What are the closed subsets of the topology generated by U?

6. Let n be a fixed natural number. Assume $U = \{V \subseteq X : |V| = n\}$. What are the closed subsets of the topology generated by U?

7. Let $U = \{V \subseteq X : X \setminus V \text{ is finite}\}$. What are the closed subsets of the topology generated by U?

8. Let A be a fixed subset of X and let $U = \{V \subseteq X : A \subseteq V\}$. What are the closed subsets of the topology generated by U?

9. Let $A \subseteq X$ and let $U = \{V \subseteq X : A \cap V = \emptyset\}$. What are the closed subsets of the topology generated by U?

10. Let $X = \mathbb{R}$ and $U_2 = \{[a, b) : a, b \in \mathbb{R}\}$. Call $\Im_2$ the topology generated by U_2. Which of the following subsets are open, closed or neighter open nor closed.

i) (-1,1) **ii)** $\mathbb{Z}$ **iii)** $\mathbb{Q}$ **iv)** $[0,1) \cup (3,4)$ **v)** (0,1]