Linear Algebra
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Do only (and only) one of the two problems.

I. Let K be a field of characteristic $\neq 2$ and n an integer. Let $V = K^n$. Let $f : V \times V \longrightarrow K$ be a nondegenerate, bilinear and symmetric form. Let $U_n(K, f) = \{g \in \mathrm{GL}_n(K) : f(v, w) = f(gv, gw)\}$. Show that if $g \in \mathrm{O}_n(K, f)$ then $\det g = \pm 1$. Let $\mathrm{SO}_n(K, f) = \{g \in \mathrm{O}_n(K, f) : \det g = 1\}$. Let $\mathrm{PSO}_n(K, f) = \mathrm{SO}_n(K, f)/Z(\mathrm{SO}_n(K, f))$.

For $K = \mathbb{C}$, $\mathbb{R}$ or a finite field F_q and various n and various f's study the groups $\mathrm{O}_n(K, f)$, $\mathrm{SO}_n(K, f)$ and $\mathrm{PSO}_n(K, f)$.

II. Classify nondegenerate, bilinear, symmetric forms over a field of characteristic 2.