Math 131 Final
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1. *How many words can you write using all the letters of* ABRAKADABRA? (A *must be used* 5 *times*, B *twice etc*.) (10 pts.)

2. *Consider the polynomial* $(X_1 + X_2 + ... + X_n)^k$ *in n variables* X_1, ..., X_n. *When multiplied out, this polynomial is equal to a polynomial of the form*

$$\sum_{i_1+i_2+...+i_n=k} a(i_1,...,i_n) X_1^{i_1} X_2^{i_2} ... X_n^{i_n}$$

for some $a(i_1, ..., i_n) \in \mathbb{N}$. *Here, k runs over all natural numbers and* i_1, i_2, ..., i_n *run over all natural numbers whose sum is k. Find* $a(i_1, ..., i_n)$. *Applying the above formula to various values of* $X_1, X_2, ..., X_n$ *deduce some combinatorial formulas*. (20 pts.)

3. *Let* f_n *be the number of words in letters a, b and c's of length n without the subword abc*.
3a. *Find a recursive formula for* f_n.
3b. *Compute* f_6 *and* f_7.
(20 pts.)

4. *How many irreducible polynomials are there in* $\mathbb{Z}[X]$ *of the form* $X^2 + aX + b$ *where a, b* $\in \{-2, -1, 0, 1, 2\}$? (15 pts.)

5. *Find all irreducible polynomials of degree* 3 *of* $(\mathbb{Z}/2\mathbb{Z})[X]$. (15 pts.)

($\mathbb{Z}/2\mathbb{Z} = \{0, 1\}$ is the ring with two elements where $1 + 1 = 0$.)