Math 232
(Linear Algebra)

Midterm 2 for CS majors
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I. Are the following sets vector spaces over $\mathbb{R}$?

a. $A = \{f : \mathbb{R} \to \mathbb{R} : f(x) \geq 0 \text{ for all } x \in \mathbb{R}\}$

b. $B = \{f : \mathbb{R} \to \mathbb{R} : f(x) = 0 \text{ for all } x \geq 0\}$

c. $C = \{(x, y, z) \in \mathbb{R}^3 : x + y + z \in \mathbb{Z}\}$

d. $D = \{(x, x^2) \in \mathbb{R}^2 : x \in \mathbb{R}\}$

Justify your answers.

II. Find a basis of the following vector space:

$$\{(2z, x - y, x - y, z + t, z - t, u + x - y) \in \mathbb{R}^6 : x + y + z + t + u = 0\}.$$

III. Find the general form of a vector of the subspace of $\mathbb{R}^4$ generated by the vectors

$$v_1 = (1, 2, 3, 1)$$
$$v_2 = (1, 2, 3, 2)$$
$$v_3 = (1, 2, 3, -1)$$

IV. Let $f : \mathbb{R}^3 \to \mathbb{R}^4$ be defined by $f(x, y, z) = (x - y, 0, 2x - 2y, x - y - 2z)$.

IV.1. Show that f is a linear map.

IV.2. Find a basis of $\text{Im}(f)$.

IV.3. Find a basis of $\text{Ker}(f)$.

V. Let $W = \{(x - y, x - y + z, z, 0, 2z) : x, y, z \in \mathbb{R}\}$. W is a subspace of $\mathbb{R}^5$. Find a basis of the quotient space $\mathbb{R}^5/W$.

VI. Let $f : V \to W$ be a linear map between two vector spaces V and W.

VI.1. Show that if $v_1, ..., v_n \in V$ are such that $f(v_1), ..., f(v_n)$ are linearly independent, then $v_1, ..., v_n$ are also linearly independent.

IV.2. Conclude that $\dim V \geq \dim f(V)$.