# Math 232
# (Linear Algebra)

Midterm 1 for CS majors

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**I.** Are the following sets vector spaces over $\mathbb{R}$?

**a.** $A = \{f : \mathbb{R} \rightarrow \mathbb{R} : f(x) \geq 0 \text{ for all } x \in \mathbb{R}\}$

**b.** $B = \{f : \mathbb{R} \rightarrow \mathbb{R} : f(x) = 0 \text{ for all } x \geq 0\}$

**c.** $C = \{(x, y, z) \in \mathbb{R}^3 : x + y + z \in \mathbb{Z}\}$

**d**. $D = \{(x, x^2) \in \mathbb{R}^2 : x \in \mathbb{R}\}$

Justify your answers.

**II.** Find a basis of the following vector space:

$$\{(x, y, x + 2y, z + t, z - t, u + x) \in \mathbb{R}^6 : x + y + z + t + u = 0\}.$$

**III.** Find the subspace of $\mathbb{R}^4$ generated by the vectors

$$v_1 = (1, 2, 3, 1)$$
$$v_2 = (1, 2, 3, 2)$$

**IV.** Let $f : \mathbb{R}^3 \rightarrow \mathbb{R}^4$ be defined by $f(x, y, z) = (x - y, 0, 2x - 2y, x + y - 2z)$.

**IV.1.** Show that $f$ is a linear map.

**IV.2.** Find a basis of $\mathrm{Im}(f)$.

**IV.3.** Find a basis of $\mathrm{Ker}(f)$.

**V.** Let $W = \{(x - y, x - y + z, z, 0, 2z) : x, y, z \in \mathbb{R}\}$. $W$ is a subspace of $\mathbb{R}^5$. Find a basis of the quotient space $\mathbb{R}^5/W$.

**VI.** Let $f : V \rightarrow W$ be a linear map between two vector spaces $V$ and $W$.

**VI.1.** Show that if $v_1, ..., v_n \in V$ are such that $f(v_1), ..., f(v_n)$ are linearly independent, then $v_1, ..., v_n$ are also linearly independent.

**IV.2.** Conclude that $\dim V \geq \dim f(V)$.

**VII.** Let $V$ be a vector space and $A$ and $B$ be two subspaces of $V$. Show that $A + B = \mathrm{Vect}(A \cup B)$.

**VIII.** Let $V$ and $W$ be two vector spaces of dimension $n$ and $m$ over the same field $K$. Show that $V \times W$ is a vector space of dimension $n + m$.