Set Theory, Second Quiz Ali Nesin December 10, 1996

1. Let x and a be two sets. Recall that $\cap x$ means the intersection of all sets which are in x, i.e. $\cap x = \{z: z \in y \text{ for all } y \in x\}.$

1a. Prove that "the object" $b := \{y \cap a : y \in x\}$ is a set. **1b.** Show that $a \cap (\cap x) = \cap b$.

2. Let *x* be a set. Show that the elements of *x* with exactly two elements form a set.

3. Cartesian Product of Two Sets. Let x and y be two sets. Show that "the object" $\{\{a\}, \{a, b\}\}: a \in x, b \in y\}$ is a set¹.

¹ This set is called the Cartesian product of *x* and *y* and is denoted by $x \times y$.