Set Theory, First Homework

(Inductive Sets)

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1. Show that if x is a set, then $x \cup \{x\}$ is also a set. The set $x \cup \{x\}$ is sometimes denoted $x + 1$.

2. A set x is called **inductive** if $\varnothing \in x$ and if for all $y \in x$, the set $y + 1$ (defined above) is also an element of x. Write a formula $\phi(v)$ in the language of set theory such that $\phi(v)$ holds for a set x if and only if x is an inductive set.

3. Let A be a set whose elements are inductive sets. Show that $\cap A$ is also an inductive set.

4. From the axioms of set theory that we have seen up to now, we cannot prove that there is an inductive set. Assuming there is an inductive set, prove that there is an inductive set (denoted by ω) which is a subset of every inductive set.

5. Do you have any idea of what ω can be?