Algebra (Math 211)
Resit

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March 3rd, 2003

Throughout G stands for a group.

1. Let $g \in G$ have order n and m an integer. What can you say about the order of g^m? (5 pts.)
2. Let $H, K \leq G$. Show that $\{HxK : x \in G\}$ is a partition of G. (3 pts.)
3. Let $H \leq G$. Show that there is a natural one to one correspondence between the left coset space of H in G and the right coset space of H in G. (3 pts.)
4. Show that the product of two groups is divisible[1] if and only if each factor is divisible. (4 pts.)
5. Show that if $G/Z(G)$ is cyclic then G is abelian. (5 pts.)
6. Find the torsion elements of

   a) $\mathbb{Q}/\mathbb{Z}$. (5 pts.)

   b) $\mathbb{R}/\mathbb{Q}$. (5 pts.)
7. Let H, $K \leq G$. Assume that for all $k \in K$, $H^k \leq H$. Show that $H^k = H$ for all $k \in K$. (5 pts.)
8. Find the isomorphism type of the group $(\mathbb{Z}/11\mathbb{Z})^*$. (5 pts.)
9. Find the isomorphism type of the group $\mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$. (8 pts.)
10. Let p be a prime.

    a. Find the group $\mathrm{Aut}(\mathbb{Z}/p\mathbb{Z})$. (5 pts.)

    b. Find $|\mathrm{Aut}(\mathbb{Z}/p\mathbb{Z} \times \mathbb{Z}/p\mathbb{Z})|$. (5 pts.)
11. Find $\mathrm{Aut}(\mathbb{Z})$. (5 pts.)
12. Find $\mathrm{Aut}(\mathbb{Q})$. (5 pts.)

[1] A group G is called divisible if for any $g \in G$ and any integer $n \geq n$ there is an $h \in G$ such that $g = h^n$.

13. Find $\mathrm{Aut}(\mathbb{Q}^*)$. (10 pts.)

14. Find $\mathrm{Aut}(\mathbb{Z} \times \mathbb{Z})$. (15 pts.)

15. Show that if G is centerless then there is an imbedding of G in $\mathrm{Aut}(G)$ (i.e. a one to one group homomorphism from G into $\mathrm{Aut}(G)$). (7 pts.)