Quiz

Math 111
Ali Nesin
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1. Show by induction on n, that for any natural number n and for any rational number x,

$$(1-x)^n \leq 1 - nx + \frac{n(n-1)}{2} x^2$$

2. Show that the subsets of $\mathbb{N}$ that contain an odd number form a set.

3. Show that every nonempty subset of $\mathbb{N}$ has a least element.

4. Show that for every rational number q, there is an integer n such that

$$n \leq q < n + 1.$$

(**Hint:** Use the previos question.)

5. Show that for any $a \in \mathbb{N}$ and $b \in \mathbb{N} \setminus \{0\}$, there are unique q and $r \in \mathbb{N}$ such that

a) $a = bq + r$

b) $0 \leq r < b$.

(**Hint:** Use the previous question).