Math 121

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Definition: If A and B are two sets, $A \,\Delta\, B$ denotes $(A \cup B) \setminus (A \cap B)$.

1. Let A, B, C be three subsets of X.
a) Show that $(A \cap C) \setminus (B \cap C) \subseteq A \setminus B$.
b) Show that $(A \cup C) \setminus (B \cup C) \subseteq A \setminus B$.
c) Show that $A^c \setminus B^c \subseteq B \setminus A$.
d) Show that $(A \setminus C) \setminus (B \setminus C) \subseteq (A \setminus B)$.
e) Show that $(A \setminus C) \setminus (A \setminus B) \subseteq (B \setminus C)$.

2. Let X be a set. For two subsets A and B of X, define the relation $A \equiv B$ by the condition "$A \,\Delta\, B$ is finite".
a) Show that this is an equivalence relation on $\wp(X)$.
b) Show that $\wp(X)/\equiv$ has only one element if X is finite.
c) Conversely show that if $\wp(X)/\equiv$ has only one element then X is finite.
d) Show that $\wp(\mathbb{N})/\equiv$ is infinite.
Show that for all $A, B, A_1, B_1 \subseteq X$, if $A \equiv A_1$ and $B \equiv B_1$, then
e) $A \cap B \equiv A_1 \cap B$. (You may use 1a)
f) $A \cap B \equiv A_1 \cap B_1$. (You may use 2e)
g) $A \cup B \equiv A_1 \cup B$. (You may use 1b)
h) $A \cup B \equiv A_1 \cup B_1$. (You may use 2g)
i) $A^c \equiv {A_1}^c$. (You may use 1c)
j) $A \setminus B \equiv A_1 \setminus B$ (You may use 1d)
k) $A \setminus B \equiv A \setminus B_1$ (You may use 1e)
l) $A \,\Delta\, B \equiv A_1 \,\Delta\, B_1$ (You may use 2j,k)

3) Everything is as above. Show that if $\wp(X)/\equiv$ is finite then X is finite and so $\wp(X)/\equiv$ has only one element. (Needs the Axiom of choice, unless the definition is the Dedekind finiteness.)