Set Theory Quiz

Math 111

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I. Let X be a set. Show that there is a set whose elements are 2-element subsets of X.

II. Let X be a set and $S \subseteq \wp(X)$. Show that there is a set whose elements are intersections of two distinct elements of S.

III. Let X be a set. A subset $\boldsymbol{T}$ of $\wp(X)$ is called a **topology** on X, if

a) $\emptyset \in \boldsymbol{T}$, $X \in \boldsymbol{T}$,

b) If $U, V \in \boldsymbol{T}$, then $U \cap V \in \boldsymbol{T}$,

c) If $\boldsymbol{S} \subseteq \boldsymbol{T}$, then $\cup \boldsymbol{S} \in \boldsymbol{T}$.

1) Show that $\{\emptyset, X\}$ is a topology on X.

2) Show that $\wp(X)$ is a topology on X.

3) Let $A \subseteq X$ be a subset of X. Show that $\{\emptyset, A, X\}$ is a topology on X.

4) Let A and B be two subsets of X. Find a finite topology on X that contains A and B.

5) Show that if $\boldsymbol{S}$ and $\boldsymbol{T}$ are topologies on X, then $\mathbf{S} \cap \boldsymbol{T}$ is also a topology on X.

6) Show that if Σ is a set of topologies on X, then $\cap\Sigma$ is also a topology on X.

7) Show that if $S \subseteq \wp(X)$, then the intersection $\boldsymbol{T}(S)$ of all topologies that contains S is the smallest topology on X that contains S.