Set Theory

Summer Homework
18th of June, 1999
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1. Let α and β be two ordinals. Show that if α and β are isomorphic as well-ordered sets, then $\alpha = \beta$.

2. Show that every set of ordinals is well-ordered by the membership relation.

3. Let X and Y be two sets. Show that either there is a one-to-one map from X into Y or from Y into X. (**Hint:** This result is false without the AC, so your proof must use AC either implicitly or explicitly).

4. A nonzero ordinal α is called a **limit ordinal**, if it has no predecessor, i.e. if there is no β such that $\beta^+ = \alpha$. Show that ω is the least limit ordinal. What is the next limit ordinal?

5. Let X and Y be sets and $f : X \to Y$ be an onto map. Show that there is a one-to-one map $g : Y \to X$ such that $f \circ g = \mathrm{Id}_Y$.

6. Recall that a nonzero ordinal α is called a **limit ordinal**, if it has no predecessor, i.e. if there is no β such that $\beta^+ = \alpha$. Show that if λ is a limit ordinal, then $\alpha + \lambda$ is a limit ordinal for all ordinals α.

7. Show that for every ordinal α, there is an ordinal β such that $\alpha < \beta$ and there is no function from α **onto** β.