Math 111

Midterm 3

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Özlem Beyarslan - Ali Nesin

1. A set X is called **complete** if every element of X is a subset of X.

1a. Give infinitely many examples of complete sets.

1b. Show that if A is a set of complete sets, then $\cap A$ and $\cup A$ are also complete.

1c. Show that if X is complete, then $X \cup \{X\}$ is complete.

1d. Let X be any set. Define $X_o = X$ and $X_{n+1} = X_n \cup (\cup X_n)$. Let $X_\omega = \bigcup_{n \in \mathrm{N}} X_n$. Assuming X_ω is a set, show that X_ω is the smallest complete set containing X.

1f. Assume $\{x\}$ is complete. What can you say about x?

2. What can you say about X if $X \cup \{X\} = X$.

3. A set X is $\in$**-connected** if for any two distinct elements x, y of X, either $x \in y$ or $y \in x$.

3a. Give infinitely many examples of $\in$-connected sets.

3b. Show that a subset of an $\in$-connected set is $\in$-connected.

3c. Show that if X is $\in$-connected, then $X \cup \{X\}$ is also $\in$-connected.

3d. Assume $\{x\}$ is $\in$-connected. What can you say about x?

4. Axiom of Regularity says that every nonempty set A has an element x such that $A \cap x = \varnothing$.

4a. Assuming the Axiom of Regularity show that no set x is a member of itself.

4b. Assuming the Axiom of Regularity show that there are no sets x and y such that $x \in y$ and $y \in x$.

4c. Assuming the Axiom of Regularity show that if $A \subseteq A \times A$, then $A = \varnothing$.