Math 111
Midterm 3
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1. A set $X$ is called complete if every element of $X$ is a subset of $X$.
   1a. Give infinitely many examples of complete sets.
   1b. Show that if $A$ is a set of complete sets, then $\bigcap A$ and $\bigcup A$ are also complete.
   1c. Show that if $X$ is complete, then $X \cup \{X\}$ is complete.
   1d. Let $X$ be any set. Define $X_0 = X$ and $X_{n+1} = X_n \cup (\bigcup X_n)$. Let $X_\omega = \bigcup_{n \in \mathbb{N}} X_n$.

Assuming $X_\omega$ is a set, show that $X_\omega$ is the smallest complete set containing $X$.
   1f. Assume $\{x\}$ is complete. What can you say about $x$?

2. What can you say about $X$ if $X \cup \{X\} = X$.

3. A set $X$ is $\in$-connected if for any two distinct elements $x, y$ of $X$, either $x \in y$ or $y \in x$.
   3a. Give infinitely many examples of $\in$-connected sets.
   3b. Show that a subset of an $\in$-connected set is $\in$-connected.
   3c. Show that if $X$ is $\in$-connected, then $X \cup \{X\}$ is also $\in$-connected.
   3d. Assume $\{x\}$ is $\in$-connected. What can you say about $x$?

4. Axiom of Regularity says that every nonempty set $A$ has an element $x$ such that $A \cap x = \emptyset$.
   4a. Assuming the Axiom of Regularity show that no set $x$ is a member of itself.
   4b. Assuming the Axiom of Regularity show that there are no sets $x$ and $y$ such that $x \in y$ and $y \in x$.
   4c. Assuming the Axiom of Regularity show that if $A \subseteq A \times A$, then $A = \emptyset$. 