1. How many words can you write using all the letters of ABRAKADABRA? (A must be used 5 times, B twice etc.) (10 pts.)

2. Consider the polynomial \((X_1 + X_2 + \ldots + X_n)^k\) in \(n\) variables \(X_1, \ldots, X_n\). When multiplied out, this polynomial is equal to a polynomial of the form

\[
\sum \prod_{i_1 + i_2 + \ldots + i_n = k} a(i_1, \ldots, i_n) X_1^{i_1} X_2^{i_2} \ldots X_n^{i_n}
\]

for some \(a(i_1, \ldots, i_n) \in \mathbb{N}\). Here, \(k\) runs over all natural numbers and \(i_1, i_2, \ldots, i_n\) run over all natural numbers whose sum is \(k\). Find \(a(i_1, \ldots, i_n)\). Applying the above formula to various values of \(X_1, X_2, \ldots, X_n\) deduce some combinatorial formulas. (20 pts.)

3. Let \(f_n\) be the number of words in letters \(a, b\) and \(c\)'s of length \(n\) without the subword \(abc\).

3a. Find a recursive formula for \(f_n\).

3b. Compute \(f_6\) and \(f_7\).  
(20 pts.)

4. How many irreducible polynomials are there in \(\mathbb{Z}[X]\) of the form \(X^2 + aX + b\) where \(a, b \in \{-2, -1, 0, 1, 2\}\)? (15 pts.)

5. Find all irreducible polynomials of degree 3 of \((\mathbb{Z}/2\mathbb{Z})[X]\). (15 pts.)

\((\mathbb{Z}/2\mathbb{Z} = \{0, 1\}\) is the ring with two elements where \(1 + 1 = 0\).\)