Math 131 Final
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1. *How many words can you write using all the letters of* ABRAKADABRA? (A *must be used* 5 *times*, B *twice etc*.) (10 pts.)

2. *Consider the polynomial* $(X_1 + X_2 + \ldots + X_n)^k$ *in* n *variables* $X_1, \ldots, X_n$. *When multiplied out, this polynomial is equal to a polynomial of the form*

$$\sum_{i_1+i_2+\ldots+i_n=k} a(i_1,\ldots,i_n) X_1^{i_1} X_2^{i_2} \ldots X_n^{i_n}$$

for some $a(i_1, \ldots, i_n) \in \mathbb{N}$. *Here*, k *runs over all natural numbers and* $i_1, i_2, \ldots, i_n$ *run over all natural numbers whose sum is* k. *Find* $a(i_1, \ldots, i_n)$. *Applying the above formula to various values of* $X_1, X_2, \ldots, X_n$ *deduce some combinatorial formulas*. (20 pts.)

3. *Let* f_n *be the number of words in letters* a, b *and* c*'s of length* n *without the subword* abc.
3a. *Find a recursive formula for* f_n.
3b. *Compute* f_6 *and* f_7.
(20 pts.)

4. *How many irreducible polynomials are there in* $\mathbb{Z}[X]$ *of the form* $X^2 + aX + b$ *where* $a, b \in \{-2, -1, 0, 1, 2\}$? (15 pts.)

5. *Find all irreducible polynomials of degree* 3 *of* $(\mathbb{Z}/2\mathbb{Z})[X]$. (15 pts.)
($\mathbb{Z}/2\mathbb{Z} = \{0, 1\}$ is the ring with two elements where $1 + 1 = 0$.)