

# Set Theory

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Explain the following “paradox”:

Elif is in an empty room with a window to the garden. Murat is outside the room, just in front of the door with infinitely many small balls numbered by natural numbers in his pocket: 0, 1, 2, 3, and so forth.

For some unknown reason, Murat throws the balls number 0 and 1 to Elif. Elif, not knowing how to react to such a strange behaviour, throws the ball number 0 from the window.

One second later, Murat, in his turn, not being able to explain Elif’s motifs to throw the ball number 0 to the garden, decides to repeat the experience: He sends the balls number 2 and 3 to Elif and waits for Elif’s move. Elif, this time annoyed, throws the ball number 1 to the garden.

Then, at the second second, Murat sends the balls number 4 and 5 to Elif. Elif, without hesitating sends the ball number 2 to the garden.

At the third second Murat sends the balls number 6 and 7 to Elif, and Elif throws the ball number 3 to the garden.

At the  $n^{\text{th}}$  second, Murat sends the balls number  $2n$  and  $2n + 1$  to Elif and Elif throws the ball number  $n$  to the garden.

Murat thinks as follows: Each second I throw two balls into the room and she throws one ball to the garden. Therefore each second, the number of balls in the room increases by 1. Therefore, at the infinity, there will be infinitely many balls in the room.

Elif thinks as follows: At the  $n^{\text{th}}$  second I will throw the ball number  $n$  from the window. So at the infinity, no ball will remain in the room.

Who is right?