A module is called **simple** if it has no nontrivial proper submodules. A module is called **semisimple** if it is a direct sum of simple modules.

1. Classify all simple \( \mathbb{Z} \)-modules. Find a \( \mathbb{Z} \)-module which is not semisimple.

2. Let \( R \) be a ring and \( N \) and \( M \) be two simple \( R \)-modules. Show that any \( R \)-module homomorphism from \( N \) into \( M \) is an isomorphism. Deduce that \( \text{End}_R(M) \) is a division ring.

3. Show that the following three conditions on a module \( M \) are equivalent:
   a) \( M \) is a direct sum of simple submodules.
   b) \( M \) is a sum of simple submodules.
   c) Every submodule \( N \) of \( M \) is a direct summand of \( M \), that is, there is a submodule \( N' \) such that \( M = N \oplus N' \).
   Note: You need Zorn’s Lemma.

4. Conclude that submodules and quotients of semisimple modules are semisimple.