MATH 311
GROUP THEORY
Resit exam
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1. Prove that any cyclic group of order p^n, where p is prime, is indecomposable into a direct product.

2. Let N be a normal subgroup of a group G. Prove that G/N is abelian iff N contains the derived subgroup G'.

3. Let H be a subgroup of a group G. Prove that if the product of any two left cosets of H is a left coset of H then H is a normal subgroup.

4. Prove that every group of order 4 is abelian.

5. Prove that, for any group G, the set $\mathrm{Inn}(G)$ of all inner automorphisms of G is a normal subgroup of the group $\mathrm{Aut}(G)$ of all automorphisms of G.

6. Prove that $S_n/A_n \simeq \mathbf{Z}_2$.