MATH 311
GROUP THEORY
Final exam
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1. Prove that any subgroup of order 25 of a group of order 100 is normal.
2. Prove that a cyclic group has no proper non-trivial pure subgroups.
3. Prove that the group $T_2(\mathbb{R})$ of all non-singular upper triangular 2×2 real matrices is 2-step solvable and not nilpotent.
4. Prove that any group of order > 2 has a non-identity automorphism.
5. Let $\bar{G}$ and G be the cartesian and direct products, respectively, of all groups $\mathbb{Z}_p$, where p runs over the primes. Prove that the subgroup G is not a direct summand in $\bar{G}$.
6. Show that the center of a non-cyclic free group is trivial.
7. Let G be the subgroup of S_5 generated by the permutations (123) and (45). Find a finite set of defining relations for G in these generators.