MATH 311
GROUP THEORY
Midterm exam
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1. Prove that a group G is abelian if $G/Z(G)$ is cyclic.
2. Prove that in any group the centalizer of any finite normal subgroup is a subgroup of finite index.
3. A group H is said to be *complete* if $Z(H) = 1$ and every automorphism of H is inner. Prove that if a group H is complete and H is a normal subgroup of a group G then G is the direct product of H and $C_G(H)$.
4. Prove that the group $\mathbb{Z}_2 \mathrm{wr} \mathbb{Z}$ has a trivial center.
5. Prove that $N(N(P)) = N(P)$ for any Sylow p-subgroup P of a finite group.