MATH 312
RING THEORY
Final exam
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1. Prove that the sum of two nilpotent right ideals is a nilpotent right ideal.
2. (a) Prove that in any ring every strictly nilpotent element is nilpotent.
   (b) Give an example of a ring in which there is a nilpotent element which is not strictly nilpotent.
   (c) Prove that in a commutative ring any nilpotent element is strictly nilpotent.
3. Prove that if M is an irreducible R-module then the ring $R/\mathrm{Ann}(M)$ is primitive.
4. Prove that an artinian ring without zero divisors is a division ring.
5. Prove that a ring R is local iff there is an ideal I of R such that R/I is a division ring.
6. Prove that any noetherian nil ring is nilpotent.