1. Let $a, b, c$ be the complex roots of the polynomial $x^3 - x - 1$. Find a polynomial over $\mathbb{Q}$ whose roots are $a^2, b^2, c^2$.

2. Find all quaternions $q$ such that $q^2 + 1 = 0$.

3. Divide $10 + 3i$ by $1 + i$ with remainder in the ring of Gaussian numbers $\mathbb{Z}[i]$. (Remember that the ring $\mathbb{Z}[i]$ is euclidian with the “degree” function $d(z) = |z|^2$.)

4. Prove that $M \otimes R \cong M$, for any commutative ring $R$ and $R$-module $M$. (Here $M \otimes R$ is the tensor product of the $R$-module $M$ and the $R$-module $R$.)

5. Let $R$ be the ring of all rational numbers $m/n$ with odd $n$.
   (a) Which elements of $R$ are invertible?
   (b) Which elements of $R$ are irreducible?
   (c) Is the ring $R$ factorial?
   (d) Is the ring $R$ a principal ideal domain?
   (e) Is the ring $R$ euclidian?
   (f) Is the polynomial $\frac{1}{3}x^5 + 2x + \frac{2}{3}$ irreducible over $R$?

6. Prove that for any finite field $F$ of prime characteristic $p$ the map $x \mapsto x^p$ is an automorphism of $F$. Give an example of a field of prime characteristic $p$ for which the map $x \mapsto x^p$ is not an automorphism.

7. Let $F$ be a finite field. Prove that $F[x]$ contains irreducible polynomials of arbitrarily high degree.