MATH 211
BASIC ALGEBRA 1
Resit exam
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1. Suppose $g^2 = 1$ for any element g of a group G. Prove that the group G is abelian.

2. Let H be a subgroup of a group G. Prove that the following are equivalent:

   (a) H is a normal subgroup of G;

   (b) the product of any two left cosets of H is a left coset of H.

3. Prove that the quotient group S_n/A_n is isomorphic to the group $(\{1, -1\}, \cdot)$. (Here S_n is the group of all permutations of n symbols, A_n is its subgroup of even permutations.)

4. Give an example which shows that in general the following it is *not* true: *if K is a normal subgroup of the group N, and N is a normal subgroup of the group G, then K is a normal subgroup of the group G.*

5. Prove that

   (a) the additive group of real numbers is isomorphic to the multiplicative group of positive real numbers,

   (b) the additive group of rational numbers is *not* isomorphic to the multiplicative group of positive rational numbers.

6. Prove that a field has no proper non-zero ideals.

7. Let I be the set of polynomials in $\mathbf{R}[x]$ with zero constant term. Prove that

   (a) I is an ideal of the ring $\mathbf{R}[x]$,

   (b) the quotient ring $\mathbf{R}[x]/I$ is isomorphic to the field $\mathbf{R}$.