MATH 211
BASIC ALGEBRA 1
Final exam
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1. Prove that any nonempty subset of a finite group closed under the group operation is a subgroup. Show that for infinite groups it is not true.
2. For which n the group S_5 has an element of order n?
3. Prove that the groups $\mathbb{Z}^n$ and $\mathbb{Z}^m$ are isomorphic only if $n = m$.
4. Prove that any group of order 4 is isomorphic to $\mathbb{Z}_4$ or $\mathbb{Z}_2 \times \mathbb{Z}_2$.
5. Prove that for finite abelian groups a converse of Lagrange's theorem holds: if G is a finite abelian group of order n then for any positive divisor d of n there is a subgroup of G of order d.
6. For which integers n the quotient ring $\mathbb{Q}[x]/(x^2 - n)\mathbb{Q}[x]$ is a field?
7. Let R be a nonzero commutative ring without ideals except for R and 0. Prove that either R is a field, or R is of prime order and $xy = 0$ for all $x, y \in R$.
8. Let p be a prime, and $\alpha \in \mathbb{C}$, $\alpha \neq 1$, $\alpha^p = 1$. Find the minimal polynomial of α over $\mathbb{Q}$.
9. Prove that the only automorphism of the field of real numbers is the identity mapping.