MATH 211
BASIC ALGEBRA 1
Midterm exam
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1. Prove that the order of any odd permutation in the group S_n is an even number.
2. Find all homomorphisms from $\mathbb{Z}_{10}$ to $\mathbb{Z}_{15}$.
3. Prove that $\mathbb{C}^* / \mathbb{C}(n) \simeq \mathbb{C}^*$.
4. Prove that the group $\mathbb{C}^*$ is the direct product of the subgroup $\mathbb{R}^{>0}$ of positive reals and the subgroup $U = \{z \in \mathbb{C} : |z| = 1\}$.
5. Let p be a prime, and G an abelian p-group. Let n be an integer coprime with p. Prove that $nG = G$. Show that the condition $\gcd(n, p) = 1$ is essential here.
6. Find permutations σ and π in the group S_4 such that $\text{order}(\sigma) = \text{order}(\pi) = 2$ and $\text{order}(\sigma\pi) = 4$.