

## MATH 211 Basic Algebra

### Problem Set 5

1. Let the orbits of  $\sigma \in S_n$  have sizes  $n_1, \dots, n_k$ . Find the order of  $\sigma$ .
2. What is the order of the permutation  $i \mapsto i + 1$  in  $\text{Sym}(\mathbb{Z})$ ?
3. Prove that in  $S_n$  the order of any odd permutation is an even number.
4. What is the order of the elements  $(-\sqrt{3} + i)/2$  and  $(3 + 4i)/5$  in  $\mathbb{C}^*$ ?
5. What is the order in the group  $T_2(\mathbb{C})$  of the elements

$$\begin{pmatrix} -1 & w \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} z & 0 \\ 0 & u \end{pmatrix},$$

where  $z, u \in \mathbb{C}^*$ ,  $w \in \mathbb{C}$ .

6. How many elements of order 6 are there in  $\mathbb{C}^*$ ,  $D_2(\mathbb{C})$ ,  $S_5$ ,  $A_5$ ?
7. Show that for any  $n, k, m \in \{0, 1, \dots, \infty\}$  in  $\text{Sym}(\mathbb{N})$  there are elements  $\sigma, \pi$  such that  $\text{order}(\sigma) = n$ ,  $\text{order}(\pi) = k$ , and  $\text{order}(\sigma\pi) = m$ .
8. Prove that in any group elements  $x$  and  $y^{-1}xy$  have the same order.
9. Prove that in any group elements  $ab$  and  $ba$  have the same order.
10. Suppose  $x, y$  have finite orders  $n, m$  in a group  $G$ , and  $xy = yx$ . Show that  $xy$  has finite order. Prove that if  $m$  and  $n$  are coprime then the order of  $xy$  is  $nm$ . Show that the condition "  $m$  and  $n$  are coprime " is essential.
11. What is the order of  $a^k$  if the order of  $a$  is  $n$ ?
12. How many subgroups are there in the cyclic group of order 100? 81?  $p^n$ , where  $p$  is prime?
13. Prove that the group  $\mathbb{Q}$  is not cyclic.
14. Find all subgroups of  $\mathbb{C}(p^\infty)$ .
15. Prove that any finite subgroup of  $\mathbb{C}^*$  is cyclic.