

MATH 211 Basic Algebra

Problem Set 3

1. Prove the the following groups are not isomorphic:
 - (a) \mathbb{Z} and \mathbb{Q} ,
 - (b) $\mathbb{Z}_{(p)}$ and $\mathbb{Z}_{(q)}$ for different primes p and q ,
 - (c) \mathbb{C}^* and \mathbb{R}^* ,
 - (d) \mathbb{R} and \mathbb{R}^* ,
 - (e) $\mathbb{C}(6)$ and S_3 ,
 - (f) $\mathbb{C}(4)$ and the subgroup $\{e, (12)(34), (13)(24), (14)(23)\}$ of S_4 (check that it is a subgroup indeed).
2. Prove that the following groups are isomorphic:
 - (a) $\text{UT}_2(\mathbb{R})$ and the additive group of reals,
 - (b) $\mathbb{C}(4)$ and the subgroup $\{e, (1234), (13)(24), (1432)\}$ of S_4 (check that it is a subgroup indeed).
3. Find all automorphisms of the groups \mathbb{Z} , \mathbb{Q} , $\mathbb{C}(12)$.
4. Find all subgroups of the groups $\mathbb{C}(p^\infty)$ and $\mathbb{C}(p^n)$.