Set Theory

Summer Midterm III
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1. Let X be a nonempty set that contains its elements as a subset, i.e. if $x \in X$ then $x \subseteq X$. Show that $\emptyset \in X$. Find such a set which is not an ordinal. (2 + 4 pts.)

2. Find a nonordinal X which is totally ordered by the membership relation $\in$.

3. Find two ordinals α and β and a nonincreasing map $f : \alpha \to \beta$ such that $f(x) < f(x^+)$ for all $x \in \alpha$.

4. Prove that the set of all finite subsets of ω is countable.

5. Show that if every countable subset of a totally ordered set X is well-ordered, then X is well-ordered.

6. Show that if α is an infinite cardinal, then $\alpha\alpha = \alpha$.