

# MATH 111

Homework

1998

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**1.** We will call a subset  $X$  of  $\mathbb{R}$  square-closed if for all  $x \in X$ ,  $x^2 \in X$  also.

**1a.** Show that  $\emptyset$  and  $\mathbb{R}$  are square closed subsets of  $\mathbb{R}$ .

**1b.** Show that if  $\Pi$  is a set of square-closed subsets of  $\mathbb{R}$ , then  $\cup \Pi$  and  $\cap \Pi$  are square closed subsets of  $\mathbb{R}$ .

**1c.** Let  $A$  be any subset of  $\mathbb{R}$ . Show that there is a smallest square-closed subset  $A^*$  that contains  $A$ .

**1d.** Let  $A$  be any subset of  $\mathbb{R}$ . Show that there is a largest square-closed subset  $A^\circ$  of  $A$ .