Notation:

\( \mathbb{N} \) = The set of natural numbers
\( \mathbb{Z} \) = The set of integers
\( \mathbb{Q} \) = The set of rational numbers
\( \mathbb{R} \) = The set of real numbers

1. Let \( a < b \) be two fixed real numbers. Find a bijection \( f_{a,b} \) between the open intervals \((0, 1)\) and \((a, b)\). What is its inverse? What is \( f_{a,b} \circ f_{c,d}^{-1} \)?

2. Find a bijection between \( \mathbb{R} \) and the open interval \((-1, 1)\).

3. Find a one-to-one map from \( \mathbb{N} \times \mathbb{N} \) into \( \mathbb{N} \).

4. Find a bijection between \( \mathbb{Z} \) and \( \mathbb{N} \).

5. Find a bijection between \( \mathbb{Q} \) and \( \mathbb{N} \).

6. Find a map \( f: \mathbb{R} \to \mathbb{R} \) such that \( \bigcap_{n \in \mathbb{N}} f^n(\mathbb{R}) = \emptyset \).

7. Show that there is no bijection between \( \mathbb{N} \) and the open (real) interval \((0, 1)\).