# MATH 111
Homework
October 16th, 1998
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**Notation:**

$\mathbb{N}$ = The set of natural numbers

$\mathbb{Z}$ = The set of integers

$\mathbb{Q}$ = The set of rational numbers

$\mathbb{R}$ = The set of real numbers

**1.** Let $a < b$ be two fixed real numbers. Find a bijection $f_{a,b}$ between the open intervals (0, 1) and $(a, b)$. What is its inverse? What is $f_{a,b} \circ f_{c,d}^{-1}$?

**2.** Find a bijection between $\mathbb{R}$ and the open interval $(-1, 1)$.

**3.** Find a one-to-one map from $\mathbb{N} \times \mathbb{N}$ into $\mathbb{N}$.

**4.** Find a bijection between $\mathbb{Z}$ and $\mathbb{N}$.

**5.** Find a bijection between $\mathbb{Q}$ and $\mathbb{N}$.

**6.** Find a map $f: \mathbb{R} \to \mathbb{R}$ such that $\bigcap_{n \in \mathbb{N}} f^n(\mathbb{R}) = \varnothing$.

**7.** Show that there is no bijection between $\mathbb{N}$ and the open (real) interval (0, 1).