All answers and claims which are worth proven should be proven.

1a. How many words composed with letters $a$ and $b$ of length $n$ and with an even number of $b$’s are there?

1b. How many words composed with letters $a$ and $b$ of length $n$ and with at least as many $b$’s as $a$’s are there?

2a. Show that $n! > 2^n$ for $n$ large enough.

2b. Show that $(x - 1)^n \geq x^n - nx^{n-1}$ for all $x > 1$.

2c. Show that if $0 < x < 1$ and $n > 0$ is a natural number, then
   \[(1 - x)^n \leq 1 - nx + n(n - 1)x^2/2.\]

3. Let $plq$ mean $p \land \neg q$.

3a. Draw the truth tables of $plq$, $plp$, $p(plq)$, $(plq)(qlp)$, $p(qlp)$.

3b. Is it true that any proposition is tautologically equivalent to a proposition whose only connective is $\land$, i.e., can we realize any truth table with propositions using only the connective $\land$?

4. Show that any any table that looks like a truth table is really the truth table of a proposition written using $\lor$, $\land$ and $\neg$ as connectives. Mathematically speaking, show that for any function $f : \{0, 1\}^n \rightarrow \{0, 1\}$ there is a proposition $\alpha(p_1, \ldots, p_n)$ of the propositional logic written with connectives $\lor$, $\land$ and $\neg$ and basic propositions $p_1, \ldots, p_n$ such that for any evaluation (i.e., function) $d : \{p_1, \ldots, p_n\} \rightarrow \{0, 1\}$, we have $d(\alpha(p_1, \ldots, p_n)) = f(d(p_1), \ldots, d(p_n))$. 