Math 131
Midterm
Fall 2004
Ali Nesin

All answers and claims which are worth proven should be proven.

1a. How many words composed with letters a and b of length n and with an even number of b's are there?
1b. How many words composed with letters a and b of length n and with at least as many b's as a's are there?

2a. Show that $n! > 2^n$ for n large enough.
2b. Show that $(x - 1)^n \geq x^n - nx^{n-1}$ for all $x > 1$.
2c. Show that if $0 < x < 1$ and $n > 0$ is a natural number, then

$$(1 - x)^n \leq 1 - nx + n(n-1)x^2/2.$$

3. Let $p|q$ mean $p \wedge \neg q$.
3a. Draw the truth tables of $p|q$, $p|p$, $p|(p|q)$, $(p|q)|(q|p)$, $p|(q|p)$.
3b. Is it true that any proposition is tautologically equivalent to a proposition whose only connective is $|$, i.e. can we realize any truth table with propositions using only the connective $|$.

4. Show that any any table that looks like a truth table is really the truth table of a proposition written using $\vee$, $\wedge$ and $\neg$ as connectives. Mathematically speaking, show that for any function $f : \{0, 1\}^n \to \{0, 1\}$ there is a proposition $\alpha(p_1, ..., p_n)$ of the propositional logic written with connectives $\vee$, $\wedge$ and $\neg$ and basic propositions $p_1, ..., p_n$ such that for any evaluation (i.e. function) $d : \{p_1, ..., p_n\} \to \{0, 1\}$, we have $d(\alpha(p_1, ..., p_n)) = f(d(p_1), ..., d(p_n))$.