1. Going back to the definitions, find all the complex numbers \( z \) where the function \( f(z) = |z|^2 \) is differentiable.

2. Prove the Cauchy-Riemann equations for an analytic function \( f \).

3. Let \( f = u + iv \) be an entire function where \( u \) and \( v \) are real valued functions on \( \mathbb{C} \). Find all functions \( w : \mathbb{C} \to \mathbb{R} \) such that the function \( u + iw \) is entire.

4. Is there a function \( v : \mathbb{C} \to \mathbb{R} \) such that the function \( f(x + iy) = \sin(x) + iv(x + iy) \) is entire? If so find it, otherwise disprove the existence.

5. Find the Laurent series of \( f(z) = \frac{(z^2 - 1)}{z} \) around \( z_0 = 0 \).

6. Prove that \( \cos^2 z + \sin^2 z = 1 \) for all \( z \in \mathbb{C} \).

7. Find the first four terms of the Laurent series of \( f(z) = e^z / \sin z \) around 0.

8. Find \( \int_{|z|=1/2} \frac{e^z - e^{-z}}{z^3} \, dz \).

9. Suppose \( f \) and \( g \) are analytic in a region \( D \). Suppose also that \( f(z)^2 = g(z)^2 \) for \( z \in D \). What can you say about \( f \) and \( g \)?

10. Suppose \( f \) and \( g \) are entire functions such that \( |f(z)| \leq |g(z)| \) for all \( z \). What can you say about the relationship of \( f \) and \( g \)?

11. Suppose \( f \) is analytic in a domain \( \Omega \) containing the unit disc and \( |f(z)| > 2 \) for all \( |z| = 1 \) and also \( f(0) = 1 \). Show that \( f \) must be equal to zero for some point in the unit disc.

12. Suppose \( f \) is any analytic function in the open unit disc. Show that there must be a sequence \( (z_n)_n \) with \( |z_n| \to 1 \) such that \( f(z_n) \) is bounded.